$$f = \frac{1}{(Z_2 - Z_1)(R_2 - R_1)} \left\{ Z_2 \left[Sinh^{-1}\left(\frac{R_2}{Z_2}\right) - Sinh^{-1}\left(\frac{R_1}{Z_2}\right) \right] - Z_1 \left[Sinh^{-1}\left(\frac{R_2}{Z_1}\right) - Sinh^{-1}\left(\frac{R_1}{Z_1}\right) \right] \right\} \tag{1}$$

$$\frac{\partial f}{\partial Z_1} = \frac{1}{Z_2 - Z_1} f + \frac{1}{(Z_2 - Z_1)(R_2 - R_1)} \left\{ Sinh^{-1}\left(\frac{R_1}{Z_1}\right) - Sinh^{-1}\left(\frac{R_2}{Z_1}\right) + \frac{R_2}{\sqrt{R_2^2 + Z_1^2}} - \frac{R_1}{\sqrt{R_2^1 + Z_1^2}} \right\} \tag{2}$$

$$\frac{\partial f}{\partial Z_2} = \frac{-1}{Z_2 - Z_1} f + \frac{-1}{(Z_2 - Z_1)(R_2 - R_1)} \left\{ Sinh^{-1}\left(\frac{R_1}{Z_2}\right) - Sinh^{-1}\left(\frac{R_2}{Z_2}\right) + \frac{R_2}{\sqrt{R_2^2 + Z_2^2}} - \frac{R_1}{\sqrt{R_2^1 + Z_2^2}} \right\} \tag{3}$$

$$\frac{\partial f}{\partial R_1} = \frac{1}{R_2 - R_1} f + \frac{1}{(Z_2 - Z_1)(R_2 - R_1)} \left\{ \frac{Z_1}{\sqrt{Z_1^2 + R_1^2}} - \frac{Z_2}{\sqrt{Z_2^2 + R_1^2}} \right\} \tag{4}$$

$$\frac{\partial f}{\partial R_2} = \frac{-1}{R_2 - R_1} f + \frac{-1}{(Z_2 - Z_1)(R_2 - R_1)} \left\{ \frac{Z_1}{\sqrt{Z_1^2 + R_2^2}} - \frac{Z_2}{\sqrt{Z_2^2 + R_2^2}} \right\} \tag{5}$$